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Exercise 1

Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{bmatrix}$ and $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{bmatrix}$
Compute each of the following:

a. α^{-1}

$$\begin{bmatrix} 2 & 1 & 3 & 5 & 4 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}$$

b. $\beta\,\alpha$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 6 & 2 & 3 & 4 & 5 \end{bmatrix}$$

c. $\alpha\,\beta$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 2 & 1 & 5 & 3 & 4 \end{bmatrix}$$

Exercise 3

Write each of the following permutations as a product of disjoint cycles:

a. (1235)(413)

$1 \longrightarrow 3 \longrightarrow 5 \longrightarrow 1$

(15)

$2 \longrightarrow 2 \longrightarrow 3$

$3 \longrightarrow 4 \longrightarrow 4$

$4 \longrightarrow 1 \longrightarrow 2$

(234)

(15)(234)

b. (13256)(23)(46512)

$1 \longrightarrow 2 \longrightarrow 3 \longrightarrow 2$

$2 \longrightarrow 4 \longrightarrow 4 \longrightarrow 4$

$3 \longrightarrow 3 \longrightarrow 2 \longrightarrow 5$

$4 \longrightarrow 6 \longrightarrow 6 \longrightarrow 1$

$5 \longrightarrow 1 \longrightarrow 1 \longrightarrow 3$

$6 \longrightarrow 5 \longrightarrow 5 \longrightarrow 6$

(124)(35)(6)

c. (12)(13)(23)(142)

$1 \longrightarrow 4 \longrightarrow 4 \longrightarrow 4 \longrightarrow 4$

$2 \longrightarrow 1 \longrightarrow 1 \longrightarrow 3 \longrightarrow 3$

$3 \longrightarrow 3 \longrightarrow 2 \longrightarrow 2 \longrightarrow 1$

$4 \longrightarrow 2 \longrightarrow 3 \longrightarrow 1 \longrightarrow 2$

(1423)

Exercise 39

In S_4, find a cyclic subgroup of order 4 and a noncyclic subgroup of order 4.
$\langle(1234)\rangle$ = {e, (1234), $(1234)^2$, $(1234)^3$}
The set $\langle(1234)\rangle$ under composition is a cyclic group of order 4 and a subgroup of S_4.
For the noncyclic subgroup of order 4:
Let S = {e, (12), (34), (12)(34)}.

Exercise 40

In S_3, find elements α and β such that $|\alpha| = 2$, $|\beta| = 2$, and $|\alpha\beta| = 3$.
S_3 = {e, (23), (12), (132), (123), (13)}
Let α = (12), β = (23)
Notice: α = 213, α^2 = 123, β = 132, β^2 = 123.
Notice also: $(\alpha\beta)$ = 312, $(\alpha\beta)^2$ = 231, and $(\alpha\beta)^3$ = 123.
Hence, α = (12) and β = (23) is a solution.