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Exercise 2

Suppose that ⟨a⟩, ⟨b⟩, and ⟨c⟩ are cyclic groups of orders 6, 8, and 20, respectively. Find all generators of ⟨a⟩, ⟨b⟩, and ⟨c⟩.

⟨a⟩	a^1, a^5
⟨b⟩	b^1, b^3, b^5, b^7
⟨c⟩	c^1, c^3, c^7, c^9, c^{11}, c^{13}, c^{17}, c^{19}

Exercise 7

Find an example of a noncyclic group, all of whose proper subgroups are cyclic.
U(8) = {1, 3, 5, 7} works.

$$\langle 1\rangle = \{1\}, \langle 3\rangle = \{3,1\}, \langle 5\rangle = \{5,1\}, \langle 7\rangle = \{7,1\}$$

Exercise 9

How many subgroups does Z_{20} have? List a generator for each of these subgroups. Suppose that G = ⟨a⟩ and |a| = 20. How many subgroups does G have? List a generator for each of these subgroups.
Six.

Z_{20}	19
Z_{10}	9
Z_5	4
Z_4	3
Z_2	1
Z_1	0

Exercise 13

In Z_{24}, find a generator for ⟨21⟩ ∩ ⟨10⟩. Suppose that |a| = 24. Find a generator for $\langle a^{21}\rangle \cap \langle a^{10}\rangle$. In general, what is a generator for the subgroup $\langle a^m\rangle \cap \langle a^n\rangle$?
⟨21⟩ = {0, 21, 18, 15, 12, 9, 6, 3}
⟨10⟩ = {0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22}
⟨21⟩ ∩ ⟨10⟩ = {0, 6, 12, 18}
Generator: 6
$\langle \mathbf{a}^{21}\rangle = \{\mathbf{a}^0, \mathbf{a}^{21}, \mathbf{a}^{18}, \mathbf{a}^{15}, \mathbf{a}^{12}, \mathbf{a}^9, \mathbf{a}^6, \mathbf{a}^3\}$
$\langle \mathbf{a}^{10}\rangle = \{\mathbf{a}^0, \mathbf{a}^{10}, \mathbf{a}^{20}, \mathbf{a}^6, \mathbf{a}^{16}, \mathbf{a}^2, \mathbf{a}^{12}, \mathbf{a}^{22}, \mathbf{a}^8, \mathbf{a}^{18}, \mathbf{a}^4, \mathbf{a}^{14}\}$
$\langle \mathbf{a}^{21}\rangle \cap \langle \mathbf{a}^{10}\rangle = \{\mathbf{a}^0, \mathbf{a}^{18}, \mathbf{a}^{12}, \mathbf{a}^6\}$
Generator for $\langle \mathbf{a}^m\rangle \cap \langle \mathbf{a}^n\rangle$ **in** $\mathbf{Z}_{24}$**:** $\mathbf{a}^{lcm(m,n)}$

Exercise 16

Complete the statement: |a| = $|a^2|$ if and only if |a| = **1 or** ∞

Exercise 32

Determine the subgroup lattice for Z_{12}. Generalize to Z_{p^2q}, where p and q are distinct primes.