Math 5358 - Midterm

1. Answer the following for the set $S = \{1, 2, 3, 4\}$

   a. How many 3-permutations of S are there?

   b. How many 3-combinations of S are there?

   c. Write two different 3-permutations of S that correspond to the same 3-combination.

   d. How many partitions of S into 3, non-empty, subsets are there?

   e. How many partitions of S into 3, non-empty, cycles are there?

2. Answer the following for the multiset $T = \{2 \cdot a, 3 \cdot b, 4 \cdot c\}$

   a. How many permutations of T are there?

   b. How many 3-combinations of T are there?

   c. How many (multi)subsets of size 4 can be formed, if each subset must contain at least one element of each type a, b, and c?

   d. What is the smallest set size that guarantees that there will be at least 2 a's, or at least 3 b's, or at least 4 c's?

3. a. Determine the coefficient of x^3 on $(x-2)^7$.

   b. Determine the coefficient of x^2y on $(x+y-2)^7$.

4. Find the general form for the solution to the recurrence relation $a_n = 8a_{n-2} - 16a_{n-4}$. (Suppose $a_n = q^n$).

5. Three different computer programs are to be run simultaneously on a multi-core processor.

   a. Determine the generating function that counts the number of ways to allocate the core usage, given that each of the three programs MUST use at least one core.

   b. Using your answer from part a. determine the number of ways the three programs can utilize 8 processing cores.

6. Answer the following for the sequence $h_n = \{0, 2, 6, 12, 20, ...\}$.

   a. Using Δ-sequences, show that h_n is an evalaution of a polynomial.

   b. Using the 0th diagonal of your answer in part a. show that $h_n = n^2 + n$.

   c. Express n^2 and n^1 as polynomials in $[n]_k$ using the Stirling numbers of the Second kind. Use this to rewrite your answer from part b.

7. Consider the set $S = \{1, 2, 3\}$.

   a. Determine the number of permutations of this set.

   b. Determine the number of permutations where $1 \to 1$.

c. Determine the number of permutations where $1 \to 1$ and $2 \to 2$.

d. Determine the number of permutations where $1 \to 1$, $2 \to 2$, and $3 \to 3$.

e. Use the principle of inclusion-exclusion to determine the number of permutations with no fixed points. (That is, $1 \nrightarrow 1$, $2 \nrightarrow 2$, and $3 \nrightarrow 3$.)